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Price-based unit commitment with decision-dependent uncertainty in hourly demand

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Abstract

The price-based unit commitment (PBUC) problem aims to optimise the power generating units' schedules to meet the system demand with the objective to maximise the generation companies' (GENCOs') profit. State-of-the-art PBUC models have taken into account exogenous uncertainties in renewable generation, demand, and price signals. This study proposes a novel PBUC problem formulation with endogenous or decision-dependent uncertainty (DDU) in the elastic portion of the demand. The proposed PBUC model is formulated as a mixed-integer non-linear programming (MINLP) problem with non-convex continuous relaxation. A concavification approach is developed to reformulate the non-convex MINLP model as an equivalent mixed-integer quadratic programming (MIQP) model whose continuous relaxation is convex. Case studies considering GENCOs owning and operating 3, 12, 19, and 40 generating units demonstrate the efficacy of the proposed DDU-aware PBUC formulation on the GENCOs' anticipated profits.

KEYWORDS

decision making, power generation scheduling, power generation dispatch, profitability, decision-dependent uncertainty

1 | INTRODUCTION

Unit commitment (UC) in electric power systems is an operational scheduling problem that accounts for the hourly response of the power supply from the generating resources to the variations in power demand over a short-term horizon into the future, typically spanning from one day to one week [1, 2]. The classical UC problem, also known as cost-based UC (CBUC), aims to minimise the cost of power generation by scheduling the status of generating units while satisfying their ramp up/down limits, minimum/maximum generating capacity, minimum up/down times, and reserve constraints (e.g., References [3, 4]). Since the CBUC problem does not commonly consider the network topology details (with some exceptions in References [5, 6]), the security-constrained UC (SCUC) is utilised by the independent system operators to clear the market taking into account the network security constraints [7]. Besides CBUC and SCUC, the other known UC problem, that is, the price-based or profit-based UC (PBUC) problem, is to maximise the profit of the power generation companies (GENCOs) highlighting the importance of price signals [8]. The objective function in a typical PBUC problem consists of a revenue function capturing the selling price contributor to the GENCOs' profit, and a similar power generation cost function as that of the CBUC and SCUC problems.

Research on the applications and solutions of the PBUC problem has been widely conducted over the past decades. With known forecasted market price inputs, the Lagrangian relaxation method was implemented to solve the PBUC problem with energy and ancillary services in Reference [1]. The study in Reference [9] proposed a mixed-integer linear programming (MILP) PBUC model considering a large number of generating units of different types. With the simultaneous consideration of power and reserve generation in the electricity market, the binary fish swarm algorithm was employed to solve the PBUC problem in Reference [10]. Reference [11] presented a genetic algorithm-based PBUC model to obtain the optimal solutions taking the energy contracts into account. The PBUC problem for a price-maker thermal generating unit was modelled as an MILP problem in References [12, 13]. Reference [14] utilised a sample average approximation approach to solve a two-stage stochastic PBUC problem with chance constraints considering the participation of wind power. Given the price quota curve, Reference [15] presented an MILP PBUC model for a price-maker participant. An optimal price bidding curve for the PBUC problem was developed with different pricing scenarios randomly generated by the Monte Carlo simulation in Reference [16]. The study in Reference [17] proposed a PBUC model for electricity storage arbitrage by accounting for the price effect of charge/discharge actions of the electricity arbitrage. A stochastic optimisation formulation for the PBUC problem was modelled in Reference [18] including a personal rapid transit system with the impact of wind energy on electricity prices. Reference [19] presented a hybrid Lagrange Relaxation-Secant-Differential Evolution method to solve the PBUC problem producing better results with less computational time. A binary firework algorithm was proposed in Reference [20] to improve the solution accuracy of the PBUC problem. The research efforts mentioned above all considered certain, often fixed and forecasted, price signals in the electricity markets with convex cost functions. References [21–23] proposed methods to find the optimal price with non-convex cost functions in the electricity markets. Reference [22] proposed a pricing scheme considering non-convex cost functions and obtained prices via applying equilibrium constraints. Reference [23] reviewed several pricing schemes in the electricity markets with non-convex cost functions. Other research efforts in References [9–15] formulated the PBUC problem with linear cost functions, while the quadratic cost function was taken into account in the CBUC objective function in References [3, 4].

The state-of-the-art models in the literature have primarily considered a deterministic model for load demand in PBUC formulations. Harnessing the demand flexibility, demand response (DR) programs are being widely approached in the recent years, taking into account the relationship between the supply and demand in electricity markets [24]. DR programs capture the changes in the electricity usage by the end-use customers from their normal consumption patterns in response to variations in the price for electricity over time [25]. In other words, when the end-use customers are provided with sufficient incentives or acceptable prices, they are willing to change (reschedule or reduce) their energy usage patterns, at times establishing a trade-off between their comfort and electricity bills [26]. By boosting the interaction and responsiveness of the customers, DR determines short-term impacts on the electricity markets, resulting in economic benefits for both customers and the electric utility company [27]. Common to the DR programs, demand elasticity is defined as the demand sensitivity with respect to the price and features a proportional change with that of the price signals [28]. That is, the demand for a commodity decreases as the price increases, defined as the elastic demand in the electricity markets [29]. Accordingly, the elasticity demand can affect the generation scheduling, where a more elastic demand was found to generally reduce the GENCOs' profits [30]. The proliferation of price-responsive demands in the electric sector highlights a motivation for the GENCOs to incorporate the demand elasticity to price signals into the day-ahead PBUC optimisation problem so as to achieve a more realistic estimation of the profits and generating unit schedules accordingly. In Reference [31], a stochastic day-ahead dispatch model considering DR was established to analyse the impact of the residential hybrid energy system on wind power utilisation. A novel retail market model with flexible and elastic price-based DR to the selling prices in Reference [32] demonstrated that the flexible price-based DR has a positive impact on reducing customers' costs. Reference [33] proposed a comprehensive high-resolution model for simulating both the elastic and automatic price responsiveness of demand to analyse the network impacts of dynamic pricing strategies. The study in Reference [34] formulated the fast-charging station deployment problem incorporating the elastic charging demand. The study in Reference [35] established a discrete time non-linear autonomous dynamical system model to capture the interaction and dynamics of the electricity prices and the total demand including the elastic sector by deriving an equilibrium. A novel heat-power trading model was presented in Reference [36] that addresses the market equilibrium of an integrated heat-power system with strategic providers and demand elasticity. The study in Reference [37] proposed a market-based control optimisation model for system operation considering the electric and social behaviours of the demand sector captured through attitude parameters in the net power injection/withdrawal. The study in Reference [38] investigated the impact of the demand price responsiveness on the oligopoly market performance by considering exogenous changes in the own-price elasticity. A hierarchical model predictive power dispatch and control strategy for modern power systems with price-elastic controllable demand was presented in Reference [39] with a suggested price-elastic utility function model. Reference [40] proposed an approach to determine the optimum level of the secondary reserve. A day-ahead scheduling problem integrating an hourly DR model for both fixed and price-elastic demand was proposed in Reference [41] to reduce the system operation costs. The study in Reference [42] presented an elastic demand scheduling model for generating a number of suitable price-based demand bids and discussed the impact of DR on the gross surplus from load serving entities.

The extant literature [9–15] on the PBUC problem ignores the decision-dependent sources of uncertainty in the decision-making process. In the traditional practice, the energy price set by the GENCOs in the PBUC model is assumed to be known and acceptable by customers, enabling GENCOs to estimate the generation profit. The uncertain volume of the elastic demand within the PBUC optimisation problem, however, depends on the corresponding selling price set by the GENCOs, which itself is the decision variable taken within the PBUC optimisation model. That is, the elastic customer may or may not accept the GENCOs' price in the market, thereby resulting in uncertain willingness-to-pay response of the elastic demand to the energy price. If not properly modelled, the inaccurate treatment of the customers' response to the GENCOs' set price may endanger the GENCOs' profit estimation with misleading generating unit schedules. Different from the existing literature, this study proposes a mixed-integer non-linear programming (MINLP) optimisation model for the PBUC problem under decision-dependent uncertainty (DDU). We model the elastic demand as an endogenous or decision-dependent source of uncertainty. A concavification approach is introduced to derive the exact representation of a product of two continuous variables and to reformulate the MINLP model as an equivalent mixed-integer quadratic programming (MIQP) model whose continuous relaxation is convex. The effectiveness of the proposed models are tested on a variety of case studies including GENCOs with 3, 12, 19, and 40 generating units.

The rest of this study is organised as follows: Section 2 introduces the DDU and gives insights on its involvement in the PBUC problem. Section 3 introduces the proposed PBUC model with DDU. Section 4 describes the method to reformulate the MINLP problem as an equivalent MIQP formulation. The numerical results and discussions are provided in Section 5, and finally Section 6 concludes the study.

2 | DECISION-DEPENDENT UNCERTAINTY

One may distinguish two main types of uncertainties: *exogenous uncertainties* and *endogenous uncertainties* or DDUs [43, 44]. In the former, the uncertainties are independent of the decisions, while in the latter, the uncertainty is impacted by the decisions taken within the optimisation problem. Within the endogenous or decision-dependent class of uncertainties, several forms of DDUs can be also distinguished (see taxonomy in Reference [43]): Type 1 DDU and Type 2 DDU. In the former, the decisions impact the probability distribution of (some) random variables, while the latter class focusses on two- or multi-stage stochastic programs with recourse in which decisions influence the time at which information is revealed and the uncertainty is resolved. In addition to the aforementioned two main classes of DDUs, some other types of DDUs are recently introduced in Reference [43], involving DDUs in robust optimisation [45] and in distributionally robust optimisation [46].

From the GENCO's perspective, which attempts to maximise profit through solving a PBUC optimisation, the customers' response to the price set for the elastic demand is uncertain and not known. The GENCO can, however, affect customers' decision to pay the price set for the elastic demand by varying the price for the elastic demand, which is one of the decisions in the PBUC problem. The willingness-to-pay response of the elastic demand, whether to accept the price set by the GENCO, is therefore a decision-dependent (endogenous) uncertainty that is affected by the pricing decisions taken by the GENCO (i.e., Type I DDU). The elastic demand is uncertain and changes over time due to the variations in the corresponding price offered by the GENCO. Figure 1 illustrates the proposed DDU mechanism in the elastic demand when integrated into the PBUC problem. When the decision maker increases the price offered to the elastic demand—see Decision 1 in Figure 1—the quantity of the elastic demand decreases, which causes a reduction in the total demand in the system. With the reduction in the electricity demand, the generating unit schedules in the PBUC problem will change accordingly (i.e., their output power may be decreased, some high-cost generating units may be turned off etc.). On the contrary, if the price for the elastic demand is reduced (Decision 2), the amount of the elastic demand will increase, indicating that a different schedule for the generating units will be decided in the PBUC problem. Therefore, the elastic portion of the demand plays a significant role in the proposed PBUC problem formulation and solutions. Accordingly, we model the elastic demand as a DDU.

The modelling of the (endogenous) dependency connecting random and decision variables is challenging and often results in the formulation of non-convex problems. To avoid the inherent modelling and solution challenges, simplifying assumptions are often used. We refer the readers to [47–51] for a detailed discussion of these simplifications and the issues they cause (i.e., models not representative of the actual problems and questionable decisions). The proper modelling of the dependency of the uncertainties on decisions relies on a *coupling function* [52]. In our model, the demand for electricity is considered to be of fixed and elastic proportions (fixed demand and elastic demand). Correspondingly, the model includes the prices for the fixed demand and the elastic demand. The price for the fixed demand is set as a parameter known day-ahead (not a decision), while the price for the elastic demand is a decision variable in our proposed PBUC model. Here, we represent the decision-dependent nature of the elastic demand d_t^e (with an upper bound of D_t^e) via a coupling function $f(\pi_t^e)$ that specifies how the decision variable π_t^e defining the price for the elastic demand impacts the uncertain quantity of the elastic demand.

$$d_t^e = f(\pi_t^e) = \frac{M^e - \pi_t^e}{K^e} \quad t \in \mathbf{T} \tag{1}$$

The coupling Equation (1) defines the elastic demand as a decreasing function of the price. The strictly positive parameters M^e and K^e denote the maximum acceptable price by customers and the slope of the elastic demand bidding curve, respectively. This indicates that the lower the price for the elastic demand, the higher its elastic volume. Note that the coupling function is deterministic and the term endogenous uncertainty or DDU can at first sound confusing. This terminology has been used for decades in the literature (see, e.g., References [43, 51]) and we use it accordingly.

3 | PROBLEM FORMULATION

In this section, we propose a new PBUC optimisation model (**PBUC-DDU**) that explicitly accounts for the DDU nature of the elastic portion of the demand whose volume is impacted by the price for the elastic demand in the electricity market. The proposed model determines the generation output and the schedules of the system generating units at each time period to maximise the GENCOs' profits. A GENCO's profit is impacted by the amount of power generation output, price, and the cost of power generation. To capture this dependence, we define the relationship between the price for the elastic demand and its volume as $\pi^e_t = M^e - K^e d^e_t$, according to the coupling Equation (1). This linear relationship indicates that the change in the price for the elastic demand results in a change in its volume. The proposed model is formulated as an MINLP problem with the following objective function:

$$\max \sum_{g\in\mathbf{G}}\sum_{t\in\mathbf{T}}\Big[\big(\delta_t p^f_{g,t} + \pi^e_t p^e_{g,t} + \rho^s_t r^s_{g,t} + \rho^n_t r^n_{g,t}\big) - \big(a_g\lambda^2_{g,t} + b_g\lambda_{g,t} + c_g x_{g,t} + C^u_g y_{g,t} + C^d_g z_{g,t}\big)\Big] \tag{2}$$

The objective function maximises the total profit through effective decision-making on the price for the elastic demand. The objective Equation (2) is composed of two terms. The first term reflects the total revenue achieved by the GENCOs for selling the generation products, including the amount of generated power supplying the elastic demand as one of the income sources. The second term indicates the total costs of power generation with a quadratic cost function, start-up cost and shut-down cost for the generating unit g [9]. The objective Equation (2) is quadratic due to the quadratic term $a_g\lambda^2_{g,t}$ in the cost function as well as the DDU in the elastic demand that requires the introduction of a bilinear term $\pi^e_t p^e_{g,t}$. The proposed optimisation model has a mixed-integer linear feasible set defined by the operation constraints (supply–demand balance, ramp rate, min-up/min-down time, and reserve) described in Subsections 3.1–3.4.

3.1 | Supply–Demand balance constraints

In this study, the GENCO serves as a price-maker entity in the market, owning and operating a number of generating units. Here, we define the DR bids consisting of bids for the hourly fixed demand and the elastic demand. Based on the forecasts of the fixed demand, the GENCO follows the price for the fixed demand to sell the generated power, where the higher the price, the lower the elastic demand. The relationship between the price for the elastic demand and its quantity is denoted by Equation (3a), which is linearly presented by References [28, 42, 53–55]. When the price set by the GENCO is below the maximum price M^e that is acceptable by the customers, customers may choose to increase their demand, on top of the fixed demand. While generic enough to accommodate a variety of price-responsiveness levels and functions for the demand across the system, the elastic demand in all four test systems is assumed to follow a linear price–demand relationship in Figure 1. Note that the coupling Equation (1) is enforced via the linking Equation (3a). In an electric power system, the power balance should hold across the system where the total generation should be equal to the total required demand. The amount of the generated power supplying the fixed demand should be equal to the quantity of the fixed demand, as shown in Equation (3b). Equation (3c) similarly indicates the power balance requirements for the elastic demand. The non-negative variable d^e_t is bounded above by D^e_t, which represents the maximum amount of the elastic demand (Equation (3d)). The sum of the generated power supplying the fixed and elastic demand represents the total amount of power output provided by system generators in Equation (3e). Equation (3f) restricts the price π^e_t from exceeding the maximum acceptable price M^e.

$$\pi^e_t = M^e - K^e d^e_t \quad t\in\mathbf{T} \tag{3a}$$

$$\sum_{g\in\mathbf{G}} p^f_{g,t} = D^f_t \quad t\in\mathbf{T} \tag{3b}$$

$$\sum_{g\in\mathbf{G}} p^e_{g,t} = d^e_t \quad t\in\mathbf{T} \tag{3c}$$

$$d^e_t \le D^e_t \quad t\in\mathbf{T} \tag{3d}$$

$$p_{g,t} = p^f_{g,t} + p^e_{g,t} \quad g\in\mathbf{G}, t\in\mathbf{T} \tag{3e}$$

$$\pi^e_t \le M^e \quad t\in\mathbf{T} \tag{3f}$$

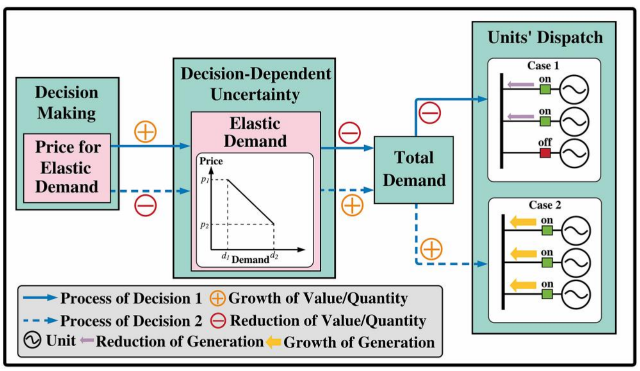

FIGURE 1 The proposed structure for the DDU-embedded profit-based unit commitment problem

3.2 | Generators' ramp rate constraints

When the binary variable $x_{g,t} = 1$, the generating unit g is online at time period t and its output power at each time period $p_{g,t}$ is required to satisfy a minimum power capacity limit $\underline{P}_g$—see Equation (4a). Equation (4b) denotes the relationship between the generating units' start-up and shut-down statuses based on their dispatch schedules. If the generating unit g is offline at time period t ($x_{g,t} = 0$) and online at the following time period $t+1$ ($x_{g,t+1} = 1$), it indicates that the generating unit g starts up at time $t+1$, which implies $y_{g,t+1} = 1$. Similarly, the generating unit g shuts down at time period $t+1$ ($z_{g,t+1} = 1$), when it is online at time period t and offline at time $t+1$. Each generating unit cannot start-up and shut-down at the same time, which is enforced by Equation (4c). Every generating unit g has its own ramp rate to restrict the amount of output power variation over time. If the generating unit g is scheduled continuously from time periods t to $t+1$ (i.e., $x_{g,t} = x_{g,t+1} = 1$, $y_{g,t+1} = z_{g,t+1} = 0$), the increase or decrease in its output power cannot be greater than the ramp-up rate α_g^+ or the ramp-down rate α_g^-, as modelled by Equations (4d) and (4e), respectively. When $y_{g,t+1} = 1$, which indicates that the generating unit g starts up at time $t+1$, the total power generated by the generating unit g at time $t+1$ cannot be greater than its start-up limit β_g^+ enforced in Equation (4d) in order to ensure the security of the generating units and the system. Similarly, when the generating unit g shuts down at time $t+1$ ($z_{g,t+1} = 1$) indicating $p_{g,t+1} = 0$, the total generation output at the previous time t is at most equal to the shut-down limit β_g^-—see Equation (4e).

$$\underline{P}_g x_{g,t} \le p_{g,t} \qquad g \in \mathbf{G}, t \in \mathbf{T} \tag{4a}$$

$$y_{g,t+1} - z_{g,t+1} = x_{g,t+1} - x_{g,t} \qquad g \in \mathbf{G}, t \in \mathbf{T} \backslash \{|\mathbf{T}|\} \tag{4b}$$

$$y_{g,t} + z_{g,t} \le 1 \qquad g \in \mathbf{G}, t \in \mathbf{T} \tag{4c}$$

$$p_{g,t+1} - p_{g,t} \le \alpha_g^+ (1 - y_{g,t+1}) + \beta_g^+ y_{g,t+1} \qquad g \in \mathbf{G}, t \in \mathbf{T} \backslash \{|\mathbf{T}|\} \tag{4d}$$

$$p_{g,t} - p_{g,t+1} \le \alpha_g^- (1 - z_{g,t+1}) + \beta_g^- z_{g,t+1} \qquad g \in \mathbf{G}, t \in \mathbf{T} \backslash \{|\mathbf{T}|\} \tag{4e}$$

$$x_{g,t} \in \{0,1\} \qquad g \in \mathbf{G}, t \in \mathbf{T} \tag{4f}$$

$$y_{g,t} \in \{0,1\} \qquad g \in \mathbf{G}, t \in \mathbf{T} \tag{4g}$$

$$z_{g,t} \in \{0,1\} \qquad g \in \mathbf{G}, t \in \mathbf{T} \tag{4h}$$

3.3 | Generators' min-up/min-down time constraints

Equations (5a) and (5b) represent the min-up and min-down time constraints [56] for the system generating units. Equation (5a) ensures that if a generating unit g turns on, it cannot be turned off immediately and needs to be kept online for at least τ_g^+ time periods. Analogously, Equation (5b) enforces that if the generating unit g turns off, it should be kept offline for at least τ_g^- time periods.

$$y_{g,t} \le x_{g,k} \qquad g \in \mathbf{G}, t \in \mathbf{T}, k \in [t, min(\tau_g^+ + t, |\mathbf{T}|)) \tag{5a}$$

$$z_{g,t} \le 1 - x_{g,k} \qquad g \in \mathbf{G}, t \in \mathbf{T}, k \in [t, min(\tau_g^- + t, |\mathbf{T}|)) \tag{5b}$$

3.4 | Reserve constraints

The operating reserve represents the total amount of generation available from all synchronised generating units in the system [2]. The operating reserve is composed of spinning reserve and non-spinning reserve. The former is the extra generation capacity that is available by increasing the power output of the already-connected generating units, while the latter is the extra generation capacity that is not currently connected to the system but can be brought online with a short delay. A generating unit g can provide spinning reserve only if it is online, while it can provide non-spinning reserve whether it is online or offline. Equation (6a) represents the total non-spinning reserve for a generating unit g consisting of the on-line and off-line non-spinning reserve capacities. The total generated power of the generating unit g is made up of its actual output power $p_{g,t}$ to meet the demand, spinning reserve $r^s_{g,t}$, and non-spinning reserve $r^n_{g,t}$ when it is online, as shown in Equation (6b). Equation (6c) specifies the upper bound of the total power generated for the generating unit g when it is online. The limitation on the operating reserve is represented by Equation (6d)–(6f), where the variables $r^s_{g,t}$, $n^+_{g,t}$, and $n^-_{g,t}$ are non-negative. Equation (6d) enforces that the total amount of reserve provided by the generating unit g cannot exceed a certain value M_g^+ when it is online [9]. Notation M_g^+ denotes the maximum sustained ramp rate for the generating unit g. Similarly, Equation (6e) determines that the amount of non-spinning reserve for the generating unit g cannot exceed its maximum quick-start capacity M_g^- when it is offline. The binary variable $h_{g,t}$ is equal to 1 if the generating unit g provides non-spinning reserve when it is offline at time period t. The generating unit g cannot be on and off at the same time, as enforced by Equation (6f). Equation (6g) stipulates that the total amount of reserve should be equal to at least the reserve requirement ratio R of the maximum generation capacity [57].

$$r^n_{g,t} = n^+_{g,t} + n^-_{g,t}, \qquad g \in \mathbf{G}, t \in \mathbf{T} \tag{6a}$$

$$\lambda_{g,t} = p_{g,t} + r^s_{g,t} + n^+_{g,t}, \qquad g \in \mathbf{G}, t \in \mathbf{T} \tag{6b}$$

$$\lambda_{g,t} \le \overline{P}_g x_{g,t}, \qquad g \in \mathbf{G}, t \in \mathbf{T} \tag{6c}$$

$$r^s_{g,t} + n^+_{g,t} \le M_g^+ x_{g,t}, \qquad g \in \mathbf{G}, t \in \mathbf{T} \tag{6d}$$

$$n^-_{g,t} \le M_g^- h_{g,t}, \qquad g \in \mathbf{G}, t \in \mathbf{T} \tag{6e}$$

$$x_{g,t} + h_{g,t} \le 1, \qquad g \in \mathbf{G}, t \in \mathbf{T} \tag{6f}$$

$$\sum_{g \in \mathbf{G}} (r^s_{g,t} + r^n_{g,t}) \ge R \sum_{g \in \mathbf{G}} \overline{P}_g, \qquad t \in \mathbf{T} \tag{6g}$$

$$h_{g,t} \in \{0,1\} \qquad g \in \mathbf{G}, t \in \mathbf{T} \tag{6h}$$

4 | REFORMULATION METHOD

We examine the form and tractability of the proposed MINLP problem **PBUC-DDU**.

Theorem 1 *The continuous relaxation of problem **PBUC-DDU***

$$\max \sum_{g\in \mathbf{G}}\sum_{t\in \mathbf{T}}\Big[\Big(\delta_t p^f_{g,t} + \pi^e_t p^e_{g,t} + \rho^s_t r^s_{g,t} + \rho^n_t r^n_{g,t}\Big) - \Big(a_g\lambda^2_{g,t} + b_g\lambda_{g,t} + c_g x_{g,t} + C^u_g y_{g,t} + C^d_g z_{g,t}\Big)\Big] \tag{7}$$
$$s.to\ (3\text{a}) - (6\text{h})$$

is non-convex.

Proof The feasible set of continuous relaxation of problem **PBUC-DDU** is linear and therefore convex. Accordingly, we only need to show that the objective function and, in particular, the non-linear function

$$f^{PBUC-DDU}_{g,t}(\pi^e_t, p^e_{g,t}) = \sum_{g\in \mathbf{G}}\sum_{t\in \mathbf{T}}\pi^e_t \cdot p^e_{g,t} \tag{8}$$

is not concave, which can be achieved by showing that its Hessian matrix

$$\mathbb{H} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

is not negative semi-definite. This follows immediately by checking the determinants of the leading principal minor determinants $\Delta_1 = 0$, $\Delta_2 = -1 < 0$, which indicates that $\mathbb{H}$ is indefinite and, in turn, that $f^{PBUC-DDU}_{g,t}(\pi^e_t, p^e_{g,t})$ is not concave. It also indicates that the continuous relaxation of **PBUC-DDU** is non-convex. □

Corollary 1 follows immediately.

Corollary 1 *The continuous relaxation of problem **PBUC-DDU** is NP-hard.*

Integer problems are typically solved with branch-and-bound or branch-and-cut algorithms that solve a continuous relaxation of the integer problem at each node on the tree. As the solution process can involve several hundreds or thousands of nodes, each involving the solution of an NP-hard continuous relaxation problem, Corollary 1 highlights the difficulty of solving the problem in its current form and the need to investigate the possible derivation of a more computationally efficient reformulation.

We shall now derive an MIQP reformulation model whose continuous relaxation is convex for problem **PBUC-DDU** whose feasible set is convex. The source of non-convexity in **PBUC-DDU** is due to the bilinear terms $\pi^e_t p^e_{g,t}$ with products of continuous variables in the objective function. Hence, we propose a concavification method for the objective function according to Equations (3a) and (3c).

Theorem 2 *Problem **R-PBUC-DDU:***

$$\max \sum_{t\in \mathbf{T}}\Bigg[\frac{M^e\pi^e_t}{K^e} - \frac{(\pi^e_t)^2}{K^e} + \sum_{g\in \mathbf{G}}\Big(\delta_t p^f_{g,t} + \rho^s_t r^s_{g,t} + \rho^n_t r^n_{g,t} - a_g\lambda^2_{g,t} - b_g\lambda_{g,t} - c_g x_{g,t} - C^u_g y_{g,t} - C^d_g z_{g,t}\Big)\Bigg]$$
$$s.to\ (3\text{a}) - (6\text{h}) \tag{9}$$

*is equivalent to problem **PBUC-DDU** and has a convex continuous reformulation.*

Proof. We first rewrite the bilinear term in the objective function of the **PBUC-DDU** problem at each time period t as $\pi^e_t \sum_{g\in\mathbf{G}} p^e_{g,t}$. By substituting d^e_t for $\sum_{g\in\mathbf{G}} p^e_{g,t}$ based on Equation (3c), and $\frac{M^e-\pi^e_t}{K^e}$ for d^e_t due to Equation (3a), we can obtain

$$\pi^e_t \cdot \sum_{g\in \mathbf{G}} p^e_{g,t} = \pi_t \cdot d^e_t = \pi_t \cdot \left(\frac{M^e - \pi^e_t}{K^e}\right) = \frac{M^e \cdot \pi^e_t}{K^e} - \frac{(\pi^e_t)^2}{K^e}\ . \tag{10}$$

Based on the new expression of the bilinear term—see Equation (10), we can replace the original objective function in **PBUC-DDU** with the reformulated objective function $f^R(\mathbb{V})$. To ease the notations, we define the set of decision variables $\mathbb{V} = \{(\pi^e_t, p^f_{g,t}, r^s_t, r^n_t, \lambda_{g,t}, x_{g,t}, y_{g,t}, z_{g,t})\colon \pi^e_t, p^f_{g,t}, r^s_t, r^n_t, \lambda_{g,t} \in \mathbf{R}_+, x_{g,t}, y_{g,t}, z_{g,t} \in \{0,1\}\}$:

$$f^R(\mathbb{V}) = \sum_{t\in \mathbf{T}}\Bigg[\frac{M^e\pi^e_t}{K^e} - \frac{(\pi^e_t)^2}{K^e} + \sum_{g\in \mathbf{G}}\Big(\delta_t p^f_{g,t} + \rho^s_t r^s_{g,t} + \rho^n_t r^n_{g,t} - a_g\lambda^2_{g,t} - b_g\lambda_{g,t} - c_g x_{g,t} - C^u_g y_{g,t} - C^d_g z_{g,t}\Big)\Bigg]\ . \tag{11}$$

The next task is now to demonstrate that the reformulated objective function $f^R(\mathbb{V})$ is concave. We can easily obtain the Hessian matrix of $f^R(\mathbb{V})$ as follows:

$$\mathbb{H} = \begin{bmatrix} -\frac{2}{K^e} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -a_g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

which is a diagonal matrix indicating that the eigenvalues of $\mathbb{H}$ are h_{ii}, where $i = 1\ldots8$. Note that the values of K^e and a_g are positive, then $h_{11} = -\frac{2}{K^e} < 0$ and $h_{55} = -a_g < 0$. Therefore, all eigenvalues of $\mathbb{H}$ are non-positive, which denotes that the matrix is negative semi-definite. With the negative semi-definite Hessian matrix, the objective function $f^R(\mathbb{V})$ is concave [58]. Note that the feasible set of continuous relaxation of the **R-PBUC-DDU** problem is linear and therefore convex. Problem **R-PBUC-DDU** is a convex MIQP model, since it is a maximisation problem of a concave quadratic objective function $f^R(\mathbb{V})$. As a result, problem **R-PBUC-DDU** is an equivalent convex reformulation of the **PBUC-DDU** problem. □

Both models **PBUC-DDU** and **R-PBUC-DDU** involve binary variables $x_{g,t}$, $y_{g,t}$, $z_{g,t}$, and $h_{g,t}$. To investigate the number of binary variables in both models, we introduce notation $n^t = |\mathbf{T}|$ to denote the number of time intervals and $n^g = |\mathbf{G}|$ to represent the number of generating units. Hence, both models have the same number of binary variables equal to $4n^t n^g$. Moreover, all constraints in both models are linear. The significant difference between the two models is that model **PBUC-DDU** owns a non-convex objective function, while the objective function in **R-PBUC-DDU** is convex. With the non-convex objective function in the **PBUC-DDU** problem, the optimisation can take a lot of time (usually exponential) to identify whether the problem is infeasible or if the solution is globally optimal. Therefore, the convex reformulation model **R-PBUC-DDU** is more efficient for solving and proving the optimality than the original non-convex model **PBUC-DDU**.

5 | NUMERICAL RESULTS AND DISCUSSIONS

In this section, numerical results are presented to verify the effectiveness of the proposed PBUC model considering the impact of DDU in the elastic demand. Through several test cases (see Subsections 5.1–5.4), we compare the performance of the proposed models with the original (and conventional) PBUC model without DDU. Subsection 5.5 analyses the computational efficiency of the proposed MINLP model **PBUC-DDU** and the equivalent MIQP reformulation model **R-PBUC-DDU**. In this study, the proposed models are tested on different GENCOs owning and operating generating units of different sizes and characteristics: 3-unit, 12-unit, 19-unit, and 40-unit GENCOs. The detailed data and information on the cost coefficients and capacity of generating units, fixed load demand at each time period etc. can be found in electronic Appendix [59]. Here, we consider that the price for the fixed demand and the market prices for the spinning and non-spinning reserves at each time period are known (forecasted) and constant in all test systems, the data on which are provided in [60]. The scheduling time horizon is set to 24 h in all tests. To investigate the impact of DDU in the elastic portion of the demand, we study two different cases for each GENCO: ***Case I***, where the PBUC problem considering DDU in the elastic demand is applied through the proposed **PBUC-DDU** model, and ***Case II***, where the traditional PBUC problem (e.g., in [8–10, 19, 20])—which is a MILP model without DDU—is utilised. In ***Case I***, the total system demand consists of the known fixed demand and the unknown quantity of the elastic demand, which is the decision-dependent source of uncertainty. All tests are conducted on a PC with an Intel Xeon E5-2620 v2 processor and 16 GB memory. The optimisation problems are formulated in AMPL and solved with the state-of-the-art optimisation solvers Baron 19.12.7 and Gurobi 9.0.2 for the MINLP problem **PBUC-DDU**, and Gurobi 9.0.2 for the MIQP problem **R-PBUC-DDU**.

5.1 | GENCO with 3 generating units

With the proposed analytics applied, Figure 2 illustrates the hourly power output of the generating units and the required demand in ***Case I***. If the generation output for a generating unit is zero at time period t, this unit is offline at time t. According to Figure 2, the generating unit $g1$ is online during the entire time horizon and is the major source of energy to satisfy the required demand. The generating unit $g2$ is online at $t = 10$–22, while the generating unit $g3$ is online only at $t = 8$, 9, 23, and 24. The hourly generation output and the required demand in ***Case II***, where the traditional PBUC model with no DDU consideration is applied, are shown in Figure 3. The generating unit $g1$ is online during the entire time horizon and offers majority of the needed energy, while the generating unit $g2$ has the same schedule as that in ***Case I***—see Figure 2. The generating unit $g3$ is online at $t = 9$, 10, 22, and 23. Comparing the total demand curves in both figures, it can be noted that the total demand curve in Figure 2 is smoother than that in Figure 3. For instance, during the off-peak time horizon (i.e., $t = 1$–8), the total demand increases when considering the DDU in the elastic portion of the demand. Such growth in demand can mitigate the variations in the demand curve during the entire time horizon. Since the elastic demand is a key feature in DR programs [28], the comparison also indicates that DR programs are able to fill the valleys for the demand curve. Additionally, the GENCO's profit in the PBUC problem in ***Case I*** is found to be $158,444, which is greater than that in ***Case II*** ($147,974). Hence, GENCOs have the opportunity to achieve additional profits when effectively capturing DDUs in the elastic demand into the PBUC optimisation problem.

5.2 | GENCO with 12 generating units

Figure 4 presents the schedule of the generating units owned and operated by the 12-unit GENCO in both ***Case I*** and

Case II. In Figure 4, different colours represent different statuses of the generating units at each time period. Here, we take *Case I* as an example. The blue colour denotes that a generating unit is online at the specific time period, while the red colour represents that the unit is offline. Generating units *g*1 and *g*6–*g*12 are online during the entire time horizon in both cases. In *Case I*, the generating unit *g*2 is always online, while unit *g*5 is always off-line. The generating unit *g*3 is online at $t = 7$–24, and the generating unit *g*4 is online at $t = 9$–21. In *Case II*, the generating unit *g*5 is offline during the entire time horizon. The generating unit *g*2 is online at $t = 10$–21, unit *g*3 is online at $t = 8$-22, and unit *g*4 is online at $t = 9$–20. Comparing the two cases, it can be noted that there are more generating units being scheduled in the off-peak time in *Case I*, where DDU in the elastic portion of the demand is captured. Moreover, the GENCO's profit using the proposed models in *Case I* ($1,725,630) is greater than that in *Case II* ($1,632,280).

5.3 | GENCO with 19 generating units

The optimal hourly generating unit schedules for the 19-unit GENCO in *Case I* and *Case II* are presented in Table 1 and Table 2, respectively. According to the results presented, the generating unit schedules are found the same in both cases. During the entire time horizon, the generating units *g*1-*g*4 and *g*7-*g*17 are always on-line, while the units *g*5 and *g*18 are always off-line. The generating units *g*6 and *g*19 are off-line only at $t = 3$-6. The GENCO's profit in *Case I* is found $3,562,990, while it is achieved as $3,483,930 in *Case II*. Consistent with the observations in other test cases, the DDU consideration in the elastic portion of demand in the PBUC problem can enhance the GENCO's profit in this test system.

5.4 | GENCO with 40 generating units

Table B-I and Table B-II presented in Appendix [59] illustrate the schedules of the system generating units owned and operated by 40-unit GENCO in both the studied test cases with and without DDU considerations. The generating units *g*6, *g*23, and *g*38 are online during the entire time horizon in *Case I* and *Case II*, while units *g*1-*g*5, *g*7-*g*17, *g*19, 20, *g*28-*g*37, *g*39, and *g*40 are offline during the entire time horizon in both cases. In *Case I*, the generating unit *g*21 is online and *g*18 is offline during the entire time horizon. Generating units *g*22 and *g*24–*g*27 switch their on/off status during the time horizon. In *Case II*, the generating unit *g*22 is online and *g*21 is off-line during the entire time horizon, while units *g*18 and *g*24–*g*27 switch their on/off status during the time horizon. The GENCO's profit in *Case I* is found to be $1,053,800, while that of *Case II* is $1,003,240, again indicating that the GENCO can achieve additional benefits via the proposed **PBUC-DDU** model.

5.5 | Computational efficiency

The proposed PBUC problem with endogenous uncertainty (or DDU) is an MINLP optimisation model whose continuous relaxation is non-convex and is not amenable to a numerical solution in its original form. The computational challenges of the considered problem are exacerbated by the integer variables and the DDU, whose modelling leads to the introduction of non-convex functions. We assess the computational efficiency of the MINLP model **PBUC-DDU** and solve the convex MIQP reformulation **R-PBUC-DDU**. The optimality tolerance level for each solver is set to 0.001%. In order to verify the efficiency and optimality of the proposed models, we designed two sensitivity analysis case studies for each GENCO: **Study I** with different selection of the quadratic term coefficient a_g in the generating units' cost function and **Study II** with different values of the maximum acceptable elastic demand D_t^e based on the base dataset available in Appendix [59]. We generated and analysed five scenarios for each study.

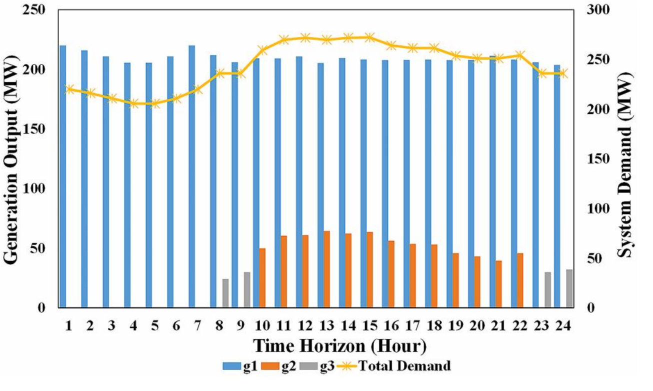

FIGURE 2 The optimal hourly generation output from the 3-unit GENCO and the required demand

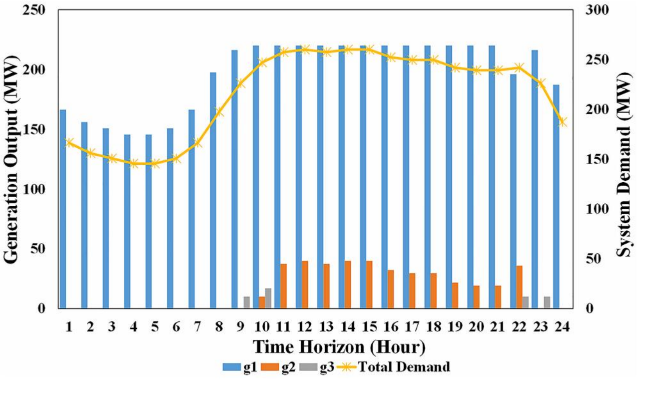

FIGURE 3 The optimal hourly generation output from the 3-unit GENCO and the required demand

5.5.1 | Study I: Different coefficients of the cost function quadratic term

Table 3 illustrates the proportion of the scenarios in **Study I**, which can be solved to optimality (tolerance level ≤0.001%) within 1 h. Based on the results presented in Table 3, one can observe that (i) none of the scenarios tested through the proposed MINLP model **PBUC-DDU** can be solved to optimality within 1 h by the Baron solver, (ii) the MINLP model could be solved to optimality by the Gurobi solver in around 80% of the scenarios with the 12-unit GENCO and none of those with the 40-unit GENCO, while all tested scenarios with the 3-unit and the 19-unit GENCOs could be solved to optimality by the Gurobi solver, and (iii) all scenarios tested with the MIQP model **R-PBUC-DDU** could be solved to optimality within 1 h. Table 4 reports the average objective values, computational times, and the optimality gaps over 5 scenarios in each GENCO for **Study I**. According to Table 4, the average objective values for both models obtained by different solvers are close to each other. The suggested MIQP model **R-PBUC-DDU** could always be solved to optimality very quickly. On the two GENCOs where both the MINLP **PBUC-DDU** and the MIQP **R-PBUC-DDU** models are solved to optimality, the average solution times over 5 scenarios is smaller for the MIQP model than that for the MINLP model (e.g., 0.03125 s vs. 0.1912 s in the 3-unit GENCO and 24.525 s vs. 274.75 s in the 19-unit GENCO).

5.5.2 | Study II: Different maximum elastic Demand levels

Table 3 presents the proportion of the scenarios in **Study II**, which can be solved to optimality within 1 h. According to Table 3, we can obtain similar observation as in **Study I**: (i) the proposed MINLP model **PBUC-DDU** tested for all scenarios cannot be solved to optimality within 1 h by the Baron solver, (ii) Applying the proposed MINLP model, all tested scenarios with the 3-unit, 12-unit, and 19-unit GENCOs can be solved to optimality by the Gurobi solver, while none of them with the 40-unit GENCO can be solved to optimality by Gurobi, and (iii) all scenarios tested with the MIQP model **R-PBUC-DDU** could be solved to optimality within 1 h. Based on Table 4, which illustrates the average objective values,

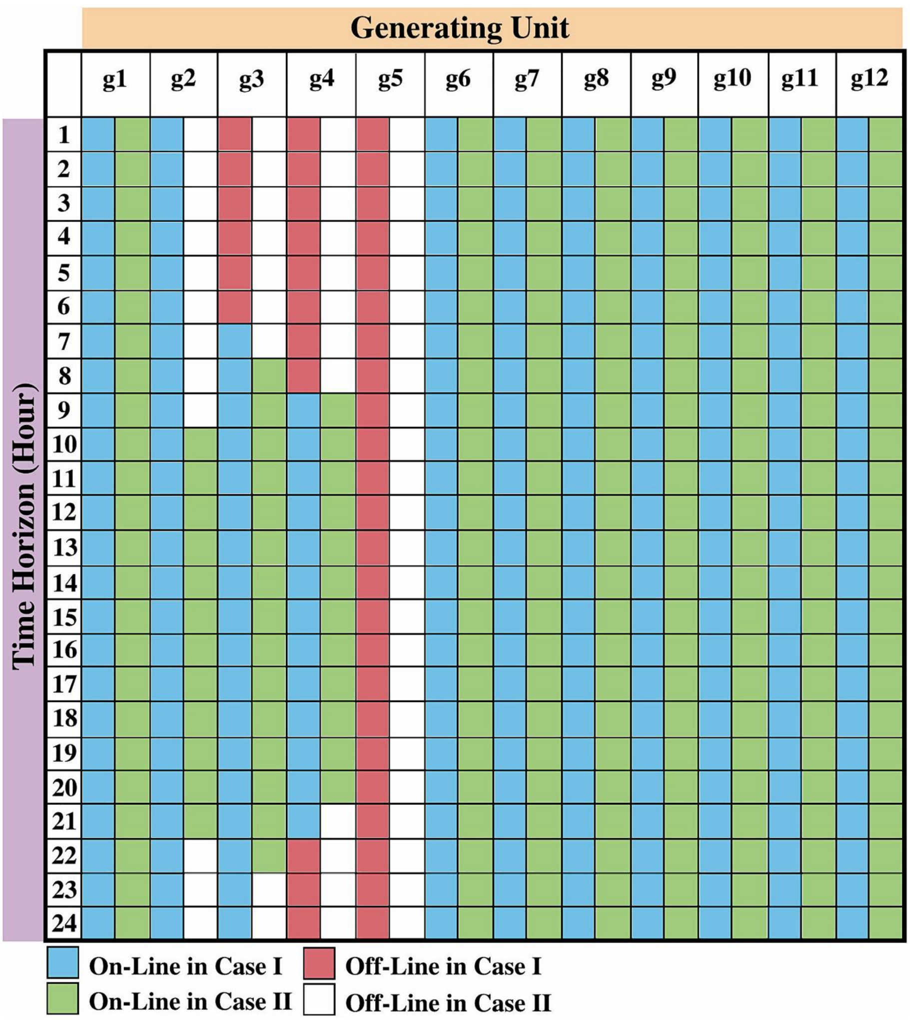

FIGURE 4 The optimal hourly schedule of generating units owned and operated by the 12-unit GENCO

TABLE 1 The optimal hourly schedule of the system generating units owned and operated by the 19-unit GENCO: ***Case I***

	Unit	Hours (1–12)											
	g1-g4	1	1	1	1	1	1	1	1	1	1	1	1
	g5	0	0	0	0	0	0	0	0	0	0	0	0
	g6	1	1	0	0	0	0	1	1	1	1	1	1
	g7–g17	1	1	1	1	1	1	1	1	1	1	1	1
	g18	0	0	0	0	0	0	0	0	0	0	0	0
	g19	1	1	0	0	0	0	1	1	1	1	1	1
Case I	**Unit**	**Hours (13–24)**											
	g1-g4	1	1	1	1	1	1	1	1	1	1	1	1
	g5	0	0	0	0	0	0	0	0	0	0	0	0
	g6	1	1	1	1	1	1	1	1	1	1	1	1
	g7–g17	1	1	1	1	1	1	1	1	1	1	1	1
	g18	0	0	0	0	0	0	0	0	0	0	0	0
	g19	1	1	1	1	1	1	1	1	1	1	1	1

TABLE 2 The optimal hourly schedule of the system generating units owned and operated by the 19-unit GENCO: ***Case II***

	Unit	Hours (1–12)											
	g1-g4	1	1	1	1	1	1	1	1	1	1	1	1
	g5	0	0	0	0	0	0	0	0	0	0	0	0
	g6	1	1	0	0	0	0	1	1	1	1	1	1
	g7–g17	1	1	1	1	1	1	1	1	1	1	1	1
	g18	0	0	0	0	0	0	0	0	0	0	0	0
	g19	1	1	0	0	0	0	1	1	1	1	1	1
Case II	**Unit**	**Hours (13–24)**											
	g1-g4	1	1	1	1	1	1	1	1	1	1	1	1
	g5	0	0	0	0	0	0	0	0	0	0	0	0
	g6	1	1	1	1	1	1	1	1	1	1	1	1
	g7–g17	1	1	1	1	1	1	1	1	1	1	1	1
	g18	0	0	0	0	0	0	0	0	0	0	0	0
	g19	1	1	1	1	1	1	1	1	1	1	1	1

computational times, and the optimality gaps over 5 scenarios in each GENCO for **Study II**, it is obvious that the suggested MIQP model **R-PBUC-DDU** could always be solved to optimality quicker than the MINLP model **PBUC-DDU** (e.g., 0.078 s vs. 0.7 s in the 3-unit GENCO, 7.94 s vs. 559.29 s in the 12-unit GENCO, and 3.28 s vs. 605.665 s in the 19-unit GENCO).

Therefore, our analyses over all the scenarios in the four GENCOs demonstrate that finding the optimal solution and proving its optimality with the convex reformulated MIQP model **R-PBUC-DDU** is significantly faster.

6 | CONCLUSIONS

This study proposed a new PBUC model to maximise the GENCOs' profits, which effectively takes into account the decision-dependent sources of uncertainty in the decision-making process. We defined the elastic portion of demand, which depends on the price for the elastic demand (a decision variable in the PBUC optimisation), as the decision-dependent source of uncertainty, that is, DDU. The proposed problem takes the form of an MINLP optimisation model. A computationally efficient concavification approach was designed to reformulate it as an equivalent convex MIQP formulation, which could be solved faster and more efficiently. Extensive numerical results on four different GENCOs clearly highlighted the effectiveness of the proposed PBUC model computationally and in increasing the GENCOs' profits when compared to the state-of-the-art PBUC models where DDUs are not incorporated. Future research could investigate different forms of non-linear relationships between the elastic demand and its corresponding price (e.g., [41], [61–63]) in the proposed PBUC model with endogenous uncertainty.

TABLE 3 Proportion of scenarios solved to the optimality tolerance level in different GENCOs

GENCO Types	PBUC-DDU		R-PBUC-DDU
	Baron	Gurobi	Gurobi
Study I			
3-Unit GENCO	0	100%	100%
12-Unit GENCO	0	80%	100%
19-Unit GENCO	0	100%	100%
40-Unit GENCO	0	0	100%
Study II			
3-Unit GENCO	0	100%	100%
12-Unit GENCO	0	100%	100%
19-Unit GENCO	0	100%	100%
40-Unit GENCO	0	0	100%

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available on from the corresponding author upon reasonable request.

NOMENCLATURE

SETS AND INDICES

$g \in \mathbf{G}$ Index and set of generating units

$t, k \in \mathbf{T}$ Index and set of time periods

PARAMETERS AND CONSTANTS

a_g Coefficient of the quadratic term for the power generation cost of generating unit g [\$/MW^2].

b_g Coefficient of the linear term for the power generation cost of generating unit g [\$/MW].

c_g Fixed cost term of generating unit g [\$].

C_g^u Start-up cost of generating unit g [\$/MW].

C_g^d Shut-down cost of generating unit g [\$/MW].

M_g^+ Maximum sustained ramp rate of generating unit g [MW].

M_g^- Maximum quick start capacity of generating unit g [MW].

$\underline{P}_g, \overline{P}_g$ Minimum and maximum power capacity of generating unit g [MW].

α_g^+, α_g^- Ramp-up and ramp-down rate of generating unit g [MW].

β_g^+, β_g^- Start-up and shut-down limit of generating unit g [MW].

τ_g^+, τ_g^- Minimum up-time and down-time of generating unit g [h].

δ_t Price for the fixed demand at time t [\$/MW].

TABLE 4 Performance comparison of the proposed mixed-integer non-linear programming and mixed-integer quadratic programming models in different GENCOs

GENCO types	PBUC-DDU Objective value (\$)		Computational time (s)		Optimality gap (%)		R-PBUC-DDU Objective value (\$)	Computational time (s)
	Baron	Gurobi	Baron	Gurobi	Baron	Gurobi	Gurobi	Gurobi
Study I								
3-Unit GENCO	154,191.6	154,191.2	3600	0.1912	0.832	0.00064	154,191.6	0.03125
12-Unit GENCO	1,917,730	1,917,729.2	3600	761.994	1.16	0.00216	1,917,729.4	0.95
19-Unit GENCO	3,125,098	3,125,124	3600	274.75	0.76	0.00084	3,125,124	24.525
40-Unit GENCO	1,031,038	1,031,004	3600	3600	4.05	0.061	1,031,344	833.185
Study II								
3-Unit GENCO	171,742.4	172,080.4	3600	0.7	1.13	0.0009	172,080.4	0.078
12-Unit GENCO	1,911,648	1,924,114	3600	559.29	2.04	0.0009	1,924,114	7.94
19-Unit GENCO	3,646,928	3,646,924	3600	605.665	1.23	0.001	3,646,924	3.28
40-Unit GENCO	1,052,848	1,053,350	3600	3600	2.8	0.564	1,053,432	1850.17

Abbreviations: DDU, decision-dependent uncertainty; PBUC, profit-based unit commitment.

ρ_t^s Market price for spinning reserve at time t [$/MW].
ρ_t^n Market price for non-spinning reserve at time t [$/MW].
D_t^f Amount of fixed demand at time t [MW].
D_t^e Maximum acceptable amount of the elastic demand at time t [MW].
M^e Maximum acceptable price for the elastic demand by customers [$/MW].
K^e Slope of the elastic demand curve [$/MW^2].
R Ratio for reserve requirement.

DECISION VARIABLES

$\lambda_{g,t}$ Total generated power including spinning and non-spinning reserve of generating unit g at time t [MW].
$n_{g,t}^+$ Non-spinning reserve of generating unit g when online at time t [MW].
$n_{g,t}^-$ Non-spinning reserve of generating unit g when offline at time t [MW].
$p_{g,t}$ Output active power of generating unit g at time t [MW].
$r_{g,t}^s$ Spinning reserve of generating unit g at time t [MW].
$r_{g,t}^n$ Non-spinning reserve of generating unit g at time t [MW].
$x_{g,t}$ Binary variable for the schedule of generating units: $= 1$ if generating unit g is on-line at time t, $= 0$ otherwise.
$h_{g,t}$ Binary variable for non-spinning supply of off-line generating units: $= 1$ if generating unit g supplies non-spinning reserve when off-line at time t, $= 0$ otherwise.
$y_{g,t}$ Binary variable for start-up status of generating units: $= 1$ if generating unit g starts up at time t, $= 0$ otherwise.
$z_{g,t}$ Binary variable for shut-down status of generating units: $= 1$ if generating unit g shuts down at time t, 0 otherwise.
$p_{g,t}^f$ Generation output of generating unit g at time t to supply the fixed demand [MW].
$p_{g,t}^e$ Generation output of generating unit g at time t to supply the elastic demand [MW].
d_t^e Quantity of elastic demand at time t [MW].
π_t^e Price for the elastic demand at time t [$/MW].

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SUPPORTING INFORMATION

Additional supporting information may be found online in the Supporting Information section at the end of this article.

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